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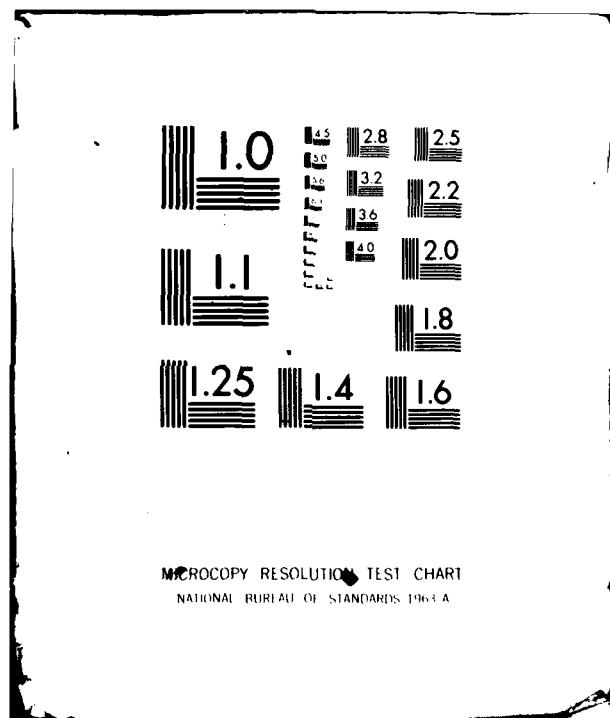
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SCATTER OF FATIGUE CRACK RATES IN 7XX ALUMINUM ALLOYS.(U)  
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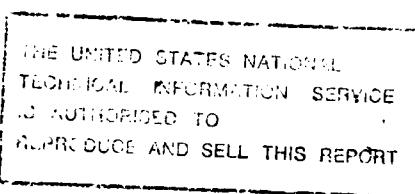
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DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION  
AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

Structures Technical Memorandum 309

SCATTER OF FATIGUE CRACK RATES IN 7XXX ALUMINIUM ALLOYS

D.G. FORD



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Structures Technical Memorandum 309

6) SCATTER OF FATIGUE CRACK RATES IN 7XXX ALUMINIUM ALLOYS

10) D.G. FORD

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SUMMARY

Thirty three crack growth curves were analysed in order to estimate the logarithmic variance of fatigue crack growth rate for several 7XXX Aluminium Alloys in various conditions. The estimated Standard Deviation common to all of these is:  $s = 0.1241$  with 18 degrees of freedom.

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### 1. INTRODUCTION

In most reliability analyses for fatigue life prediction the form of crack growth is deterministic in that there is no randomness in the assumed length of any crack that has grown for a given number of cycles. This allowance1 now has been made in recent work and the associated computer programmes.

However there is little convenient data about rate scatter known to the author in the literature. Although there is data that can be used it is inevitably in graphical form which would entail great effort for any statistical analysis. Some of this, referring to d6ac steel, has been analysed by Jost and Esson<sup>2</sup>.

For the 7XXX series of aluminium alloys the author has recently studied an Alcoa report<sup>3</sup> containing unequivocal data of a manageable form and size (33 specimens) covering a wide range of conditions. Although the amount of useable data is still small, present circumstances justify its analysis for scatter.

### 2. DATA

This consists of plotted crack growth between 1.0 and 3.5 cm in flat tension specimens 102 mm x 6.4 mm in cross section. For each specimen two crack growth curves were supplied for growth from either side of a slit. It was found that these usually coincided. This is an important result in itself because it indicates that scatter in crack growth arises mostly from differences between specimens and not during the growth of a particular crack.

Replication among this data is never more than 4 and usually 2. The main comparisons, apart from microstructure are between 7010, 7050, 7075 and 7475 alloys in T6 and T7 tempers. These are confounded with differing load programmes, the majority being constant amplitude with added overload cycles. Table 1 shows the data finally used together with Figure numbers from Reference 3 and distinctive features of each test. The unit of life, centimetres, is explained below. The results from Figures A12 and A30 are taken at a crack length of 2.28 cm.

### 3. ANALYSIS

The growth rate of any crack depends upon local stress intensity and other influences which may be grouped under the heading of material parameters. The coincidence of crack growth curves mentioned above indicates that the latter are constants for any particular specimen.

Perusal of most data for  $da/dN$  vs  $\Delta K$  on logarithmic scales indicates that variance of  $\log(da/dN)$  is sensibly constant while its average follows the well known sigmoidal curve. Since independence of mean and variance is best characterised by the Normal distribution this suggests that  $\log(da/dN)$  is Normally distributed; to assess airworthiness one should estimate this logarithmic variance. In these conditions it can be proved that the number of cycles or programme blocks applied during the time a crack grows between two lengths is also log-Normally distributed with the same variance. Accordingly for each specimen the cycles or number of programmes occupied in crack growth from 1.0 cm to 3.5 cm were measured from the relevant figure by a centimetre rule. Some typical figures and a measurement are reproduced here. Because the graphs were standardised and variation of log-times were the only quantities of interest the measured distance could be used directly. This was allowed by the fact that different scales corresponded to significant differences<sup>3</sup> caused by alloy specification, temper or load programme. In terms of analysis of variance the estimators of error variance are not affected even though calculated block or treatment effects may become quite meaningless. Differences due to deliberate changes in microstructure were taken as errors if the alloy-temper-load designation remained the same.

### 3.1 Computations

Data from Table I was first grouped, as in Table II, into the experimental designs shown. Analyses of variance<sup>4</sup> on logarithms of each of these then supplied four error sums of squares. Being compatible, these were pooled. The significance of treatment and block effects confirmed the results of Reference 3. In Table II the treatments and blocks are indicated by referring to the first entries from Table I.

The appendix illustrates the analysis of the non-orthogonal design from the first two-way classification in Table II. Crout's method has been used, adapted to a pseudo-inversion to cope with the reduced rank of the model.

### 4. CONCLUSIONS

The estimated logarithmic standard deviation from the useable data in Reference 3 is:  $s = 0.1241$  with 18 degrees of freedom. This estimate holds for differing types of 7XXX aluminium alloys, constant amplitude and block programme loading. The errors it summarises include effects of microstructure, heat treatment and purity not described by the alloy specification.

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Report ARL/SM 349, April 1974.
3. T.H. Sanders Jnr. and R.R. Sawtell et al.      Effect of Microstructure on Fatigue Crack Growth of 7XXX Aluminium Alloys under Constant Amplitude and Spectrum Loading.  
Alcoa NADC Contract N00019-76-C-0482, April 1978.
4. Oscar Kempthorne      The Design and Analysis of Experiments.  
John Wiley, 1952.

TABLE I  
DATA USED FROM REFERENCE 3

Figure	Life (cm)	Alloy	Load Type	Remarks
A7	9.9	7010-T7	CA	HI PURITY
A8	10.5	7475-T7	CA	
A9	11.2	7475-T7	CA	HI Cu
A10	8.5	7010-T7	CA	
A11	12.8	7050-T7	CA	
A12	17.2*	7075-T6	OLR 1.8	LO PURITY
	18.1*		OCR 4000	
	19.4*			
A13	11.2	7075-T7	CA	HI Cu
A14	13.7	7050-T7	CA	
A16	9.5	7075-T7	CA	
A17	3.7	7075-T6	CA	
A20	9.5	7075-T7	OLR 1.4 OCR 4000	
A21	3.8	7075-T6	OLR 1.4 OCR 4000	
A26	8.6	7050-T7	OLR 1.8 OCR 4000	
A27	10.2	7050-T7	OLR 1.8 OCR 4000	
A30	17.7*	7075-T6	OLR 1.8 OCR 4000	
A31	0.91	7010-T7	OLR 1.8 OCR 8000	HI PURITY
A32	3.4	7475-T7	OLR 1.8 OCR 8000	
A33	1.6	7475-T7	OLR 1.8 OCR 8000	HI Cu
A34	2.75	7010-T7	OLR 1.8 OCR 8000	
A35	3.20	7050-T7	OLR 1.8 OCR 8000	LO PURITY
A36	4.0	7075-T7	OLR 1.8 OCR 8000	HI Cu
A37	2.55	7050-T7	OLR 1.8 OCR 8000	
A39	5.4	7075-T7	OLR 1.8 OCR 8000	
A40	4.3	7075-T6	OLR 1.8 OCR 8000	
A41	3.1	7075-T6	OLR 1.8 OCR 8000	
A42	3.1	7475-T7	OLR 1.8 OCR 8000	
A43	3.2	7475-T7	OLR 1.8 OCR 8000	
A44	19.2	7050-T7	LHPL	
A45	15.0	7050-T6	LHPL	
A46	18.2	7075-T7	LHPL	
A47	11.6	7075-T6	LHPL	

LOAD TYPES:

CA Constant Amplitude  
 OCR Occurrence ratio of 1 overload per 4000 or 8000 cycles.  
 OLR Overload ratio 1.4, 18  
 LHPL Lo-Hi-Lo Programme Loading.  
 \* At a crack length 2.28 cm.

TABLE II  
 EXPERIMENTAL DESIGNS FOR ANALYSIS  
 Rearrangement of data from Table I

ONE WAY CLASSIFICATIONS

CASE:	A7(a)	A8	A11	A26	A31	A35
Lives	9.9	10.5	12.8	8.6	0.91	3.20
cm	8.5	11.2	13.7	10.2	2.75	2.55

Logarithmic standard deviation  $s = 0.1449$  (6 d.f.)

CASE:	A12(a)	A32
Lives	17.2	3.4
cm	18.1	1.6
	19.4	3.1
	17.7 *	3.2

$s = 0.1097$  (6 d.f.)

(a) First entry in Table I

\* Measured at a crack length 2.28 cm.

TWO WAY CLASSIFICATIONS (WITHOUT INTERACTION)

7075

Temper	Load Programme		
	CA	1.4/4000	1.8/8000
T6	3.7	3.8	4.3, 3.1
T7	9.5, 11.2	9.5	4.0, 5.4

$s = 0.1243$  (5 d.f.)

A17, A21, A40, A41, A16, A13, A20, A36, A39 by rows.

Lo-Hi-Lo Programme Load

Alloy	Temper	
	T7	T6
7050	19.2	15.0
7075	18.2	11.6

$s = 0.0442$  (1 d.f.)

A44 to A47 by rows

POOLING

Degrees of freedom =  $6 + 6 + 5 + 1 = 18$

Pooled sums of squared errors = 0.1259 6789 + 0.0721 8733 + 0.0769 8458  
 + 0.0019 5379

Variance estimate = 0.2770 9359/18 = 0.0153 9409

$s = 0.1240 7291$  (18 d.f.)

APPENDIX

ANALYSIS OF NON-ORTHOGONAL DATA

	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>
T <sub>6</sub>	3.7	3.8	4.3, 3.1
T <sub>7</sub>	9.5, 11.2	9.5	4.0, 5.4

B<sub>1</sub> - Constant amplitude

B<sub>2</sub> - OLR 1.4

B<sub>3</sub> - OLR 1.8

The table above reproduces the data in question from Table 2. As shown, it may be explained by the model,

$$y_{ijk} = T_i = B_j + e_{ijk}$$

where the y's are logarithmic data and the usual overall mean has been subsumed into the other parameters. Nevertheless there are still six parameters for a model of rank five so that the normal equations are singular. Since the lack of orthogonality precludes the usual analysis we shall solve the normal equations directly with allowance for the singularity. This amounts to using the pseudo-inverse of the coefficient matrix.

Each of the entries in the design can be explained by one of the equations

$$\begin{array}{ccccc} T_6 & T_7 & B_1 & B_2 & B_3 \\ \left[ \begin{array}{ccccc} 1 & . & 1 & . & . \\ 1 & . & . & 1 & . \\ 1 & . & . & . & 1 \\ 1 & . & . & . & 1 \\ . & 1 & 1 & . & . \\ . & 1 & 1 & . & . \\ . & 1 & . & 1 & . \\ . & 1 & . & . & 1 \\ . & 1 & . & . & 1 \end{array} \right] & = \log \left[ \begin{array}{c} T_6 \\ T_7 \\ B_1 \\ B_2 \\ B_3 \end{array} \right] & = \log \left[ \begin{array}{c} 3.7 \\ 3.8 \\ 4.3 \\ 3.1 \\ 9.5 \\ 11.2 \\ 9.5 \\ 4.0 \\ 5.4 \end{array} \right] \end{array}$$

(b)

APPENDIX (CONTD.)

or

$$\underline{x}\underline{\beta} = \underline{y}$$

which are simply the model applied to each of the entries in the data.

Premultiplication by  $\underline{x}^t$  provides the normal equations

					<u>Sum check</u>
4	.	1	1	2	T <sub>6</sub> 2.2728 10.2728
.	5	2	1	2	T <sub>7</sub> 4.3391 14.3391
1	2	3	.	.	B <sub>1</sub> 2.5951 8.5951
1	1	.	2	.	B <sub>2</sub> 1.5575 5.5575
2	2	.	.	4	B <sub>3</sub> 2.4593 10.4593

Next, the triangular factors provided by Crout's method and the corresponding right hand side combine in the auxiliary matrix

4.0	.	.25	.25	.5	0.5682 0387	2.5682 0387
.	5.0	.4	.2	.4	0.8678 2380	2.8678 2380
1.0	2.0	<u>1.95</u>	<u>-.333</u>	<u>-.666</u>	0.1493 8045	.1493 8045
1.0	1.0	<u>-.65</u>	<u>1.333</u>	<u>-1.0</u>	0.1639 3262	.1639 3262
2.0	2.0	<u>-1.3</u>	<u>-1.333</u>	<u>0</u>	0/0 use zero	<u>-2.9/0 X 10<sup>-8</sup></u> use 1.0

in which the pivot 0 ultimately reflects the linear dependence of the model through the fact that the  $B_3$  column of  $X$  depends on the first four columns.

In the back solution the solution for  $B_3$  is arbitrary and for convenience here we use  $B_3 = 0$ . The general solution may be found by adding another back solution column starting from  $B_3 = 1.0$  and combining the two with  $B_3$  as an arbitrary coefficient of the second set of solutions.

As in fully ranked models the general linear hypothesis indicates that the regression sum of squares =  $\underline{\beta}^t (\underline{x}^t \underline{y})$ , the sum of fitted parameters multiplied by their respective right members in the normal equations. Here this is more conveniently obtained from

(c)

APPENDIX (CONTD.)

the pivots and right member of the auxiliary equation in the typical form

$$R_{SS} = \underline{4.0}(.5682)^2 + \underline{5.0}(.8678)^2 + \dots \underline{0}(0)^2 \\ = 5.1363 5845$$

on four degrees of freedom, the rank of the model. This of course includes the effect of the overall mean

$$C = (\sum y_{ijk})^2 / \text{Total number} = 4.8575 1969 (1 \text{ d.f.})$$

and the difference between these numbers are marginal  $T_i$  or  $B_j$  effects of amount

$$R_{TB} = 5.14 - 4.86 = 0.2788 3876; (3 \text{ d.f.})$$

further reduction needs other techniques<sup>4</sup>. One of these take advantage of the elimination inherent in the auxiliary matrix.

From the last three (i.e. 2) rows

$$\text{Block SS} = \underline{1.95}(.1493)^2 + \underline{1.33}(.1639)^2 \\ = 0.0793 4516; (2 \text{ d.f.})$$

The first two rows similarly provide  $T_i$  and overall mean squares

$$\text{Treatment SS} + C = 5.0570 1324; (1 \text{ d.f.})$$

Consistency with,

$$R_{TB} = \text{Block SS} + \text{Treatment SS}$$

checks the arithmetic of this stage.

For the error,

$$E_{SS} = \sum y_{ijk}^2 - C \\ = 5.2133 4303 - 4.8575 1969 \\ = 0.0769 8458; (5 \text{ d.f.})$$

corresponding to the variance estimate 0.0153 9692 or

$$s = 0.1240 8435$$

on five degrees of freedom.

(d)

APPENDIX (CONTD.)

It is now convenient to display these results in the ANOVA.

<u>Source</u>	<u>Degrees of freedom</u>	<u>Sum of squares</u>	<u>Mean square</u>	<u>F</u>
Temper	1	0.1994 9355	Same	12.96 (2.5%)
Blocks, $B_i$	2	0.0793 4516		2.58 (20%)
$R_{TB}$	3	0.2788 3876		
Mean C	1	4.8575 1969		
Error	5	0.0769 3458	0.0153 9692	
<u>TOTAL</u>	<u>9</u>	<u>5.2133 4303</u>		

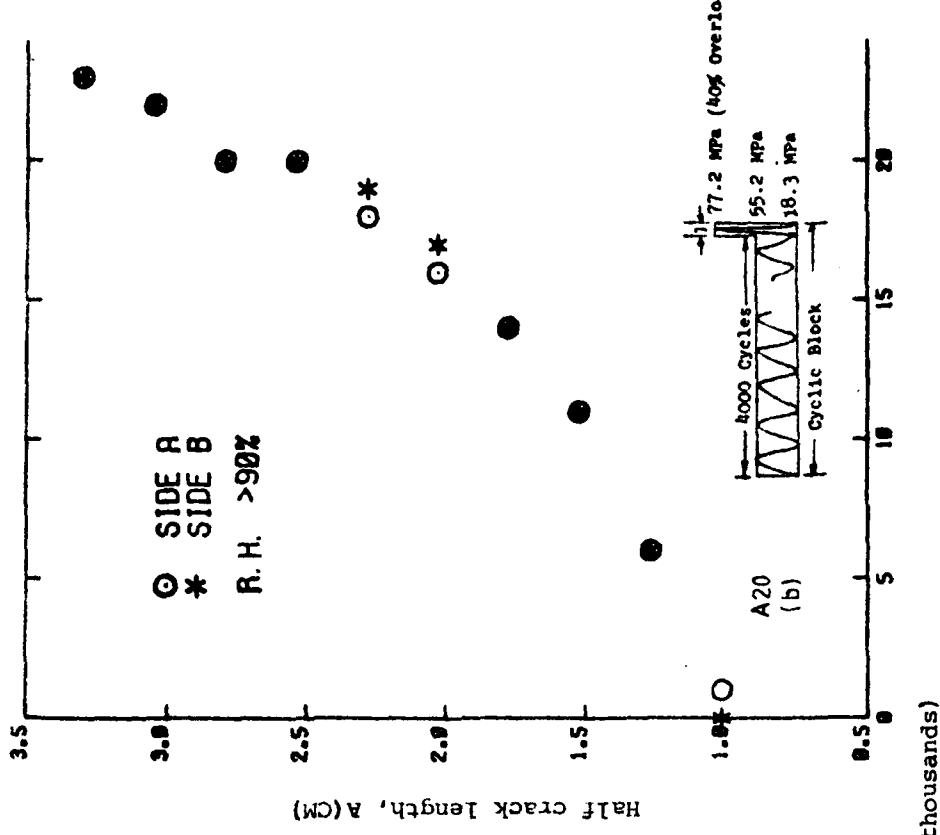
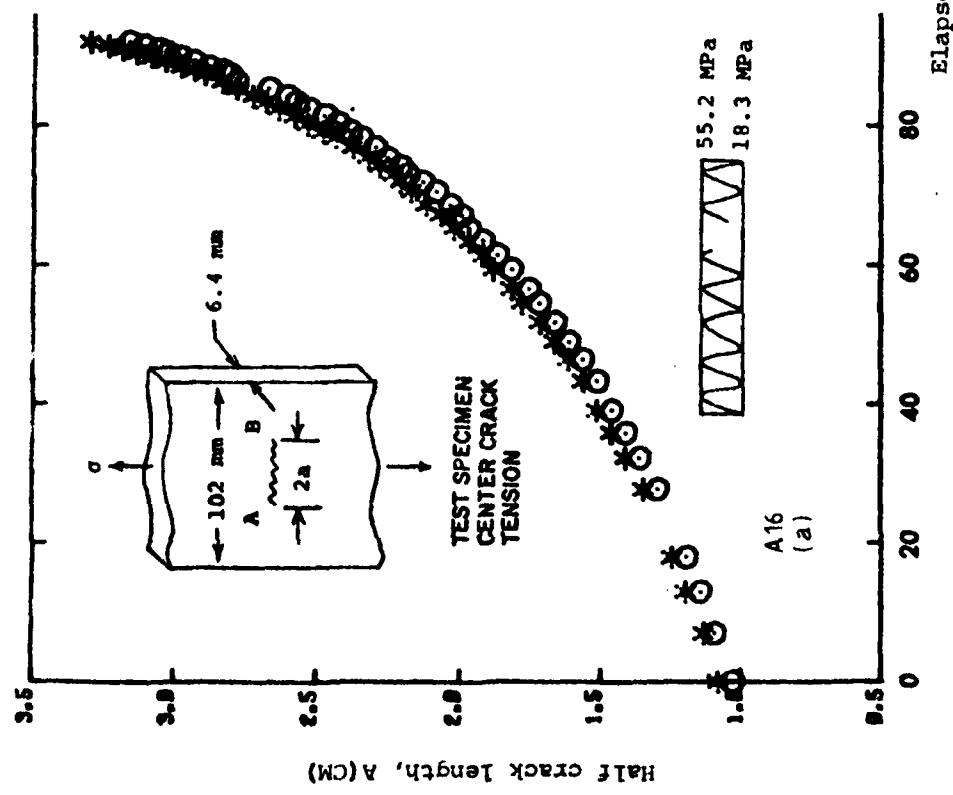


FIG. 1 EXAMPLES OF CURVES USED FOR CRACK MEASUREMENTS

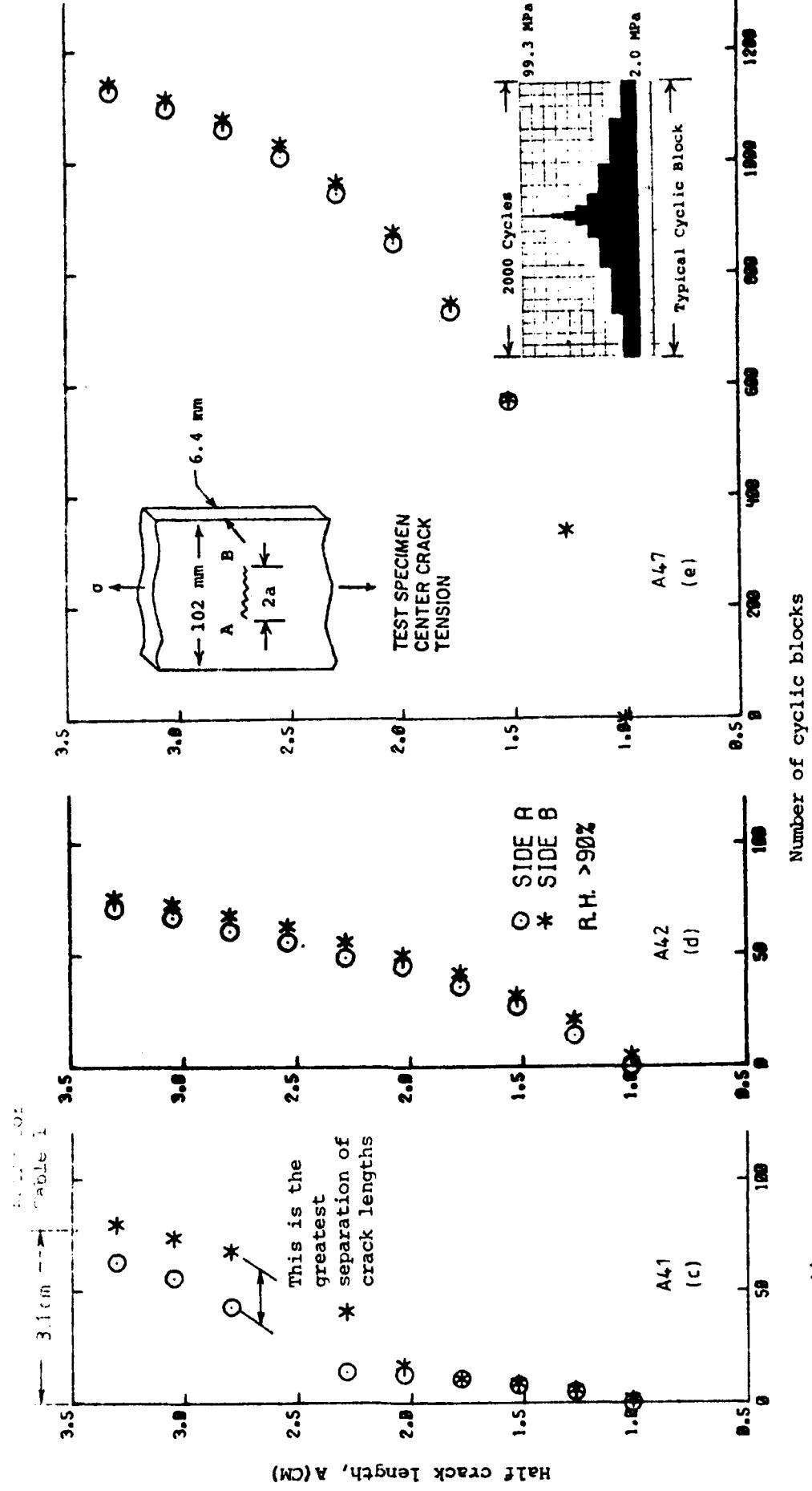


FIG. 1 EXAMPLES OF CURVES USED FOR CRACK MEASUREMENTS

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